A ROBUST MARKOV-LIKE MODEL FOR THREE PHASE ARC FURNACES

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Abstract: -- A novel approach of modeling a nonlinear, highly time varying load such as an Electric Arc Furnace (EAF) is presented in this paper. First and second order Markov-like models are formulated to compare their effectiveness in the evaluation of the stationary nature of the process and the possibility of predicting the state variable such as arc current at least one step in advance. It is seen that the statistical behavior of the EAF data is stationary with respect to time by comparing certain characteristics of the two data sets derived from the empirical frequency distributions. A second order Markov-like model is proved to be very effective in the prediction of EAF current to a good degree of accuracy. The predictor is the most probable value of the immediate future, given the present and the immediate past for each step of prediction.

Keywords: Electric Arc Furnace, Markov-like Models, Load Model, Transition Matrix, Prediction

1. INTRODUCTION

The Electric Arc Furnace (EAF) is very common in the steel manufacturing industry to melt scrap and pre-reduced metals. Since it is a large, highly unbalanced, nonlinear and time varying load, the influence of EAF on power quality is of great concern to power systems engineers. The fact that arc-length is time variant and the movement of scrap is random, it makes the current waveform look erratic. Voltage fluctuation, which is usually associated with a voltage flicker, is caused by the EAF current. For these reasons, the arcing process is assumed to be statistical in nature [1]. As a consequence, the EAF load can not be adequately represented by a deterministic dynamic model.

The EAF is such an electrically chaotic load in nature that accurate modeling of it is necessary to evaluate and mitigate its deleterious impact on a power system. In fact, a lot of work has been reported in this area. The approaches using related v-i characteristics, in which arc length, arc voltage and arc current are expressed by empirical formulas, were presented in [3,4]. The authors of ref. [5] proposed a flicker compensation technique using stochastic and sinusoidal time varying laws. The EAF current is also considered as a deterministic chaotic system by Tan et al. al [6]. Most of these models are in time domain while the authors of ref. [7] employed a frequency domain method to analyze the harmonic EAF current. Varadan et al. al [8] suggested that a single phase arc furnace model should be adequate to represent a three phase EAF circuit.

An alternative and new approach proposed in this paper is to model the EAF system as a Markov-like sequence based on ref. [9,10]. It is shown that this approach makes it possible to simulate the EAF behavior accurately. Developments in statistics and applied probability suggest that a suitable Markov-like model may fit a wide range of discrete-valued time series very well, in particular for dynamic data [9,10,11,12]. Thus the observed and recorded waveforms of an EAF system and other phenomena can be seen as nonlinear, dynamic time series, which may behave chaotically. In the following sections, the procedures for developing a first order and second order Markov-like models from field data in time series format are described. Nomenclature is defined as well. Then, the two models are evaluated by comparing the simulation results with actual current data. The paper also discusses the effectiveness of the one-step-ahead prediction approach using the results derived from the second order Markov-like modeling. The quality of the prediction is tested by comparison with real data. Simple statistical analysis is also used for this purpose.

2. MODEL DEVELOPMENT

The development of the proposed models is based on ideas from Markov's theory [9,10,12,13,14]. The application of a first order Markov-like model is discussed at first in detail. Then these ideas are extended to a second order Markov-like model. The formulation of the model involves steps i) to viii) in the following in relation to Fig. 1, which represents a typical arc current waveform.

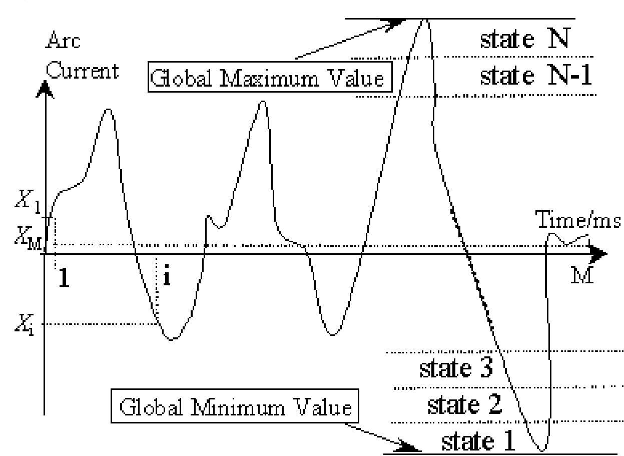

Fig. 1 Illustration of states used in Markov-like model

Step i) The global minimum and maximum values of the variable (arc current or voltage, etc.) under study over the interval of observation are identified.

Step ii) All the values of the data from the global minimum to maximum are divided into a finite number (say N) of intervals such as state 1, state 2, … state N. (see Fig. 1).

Step iii) Given the time series $\{X_j: j=1, 2,\dots M\}$ with its state space S={1, 2, …. N}, define

$\delta_{kj} = 1$ if $X_j \in$ state k

$\delta_{kj} = 0$ if $X_j \notin$ state k (1)

Then the frequency of visits to state k called Empirical Frequency Function (EFF) is

$$\pi_k = \frac{1}{M}\sum_{j=1}^{M}\delta_{kj} \tag{2}$$

Also define the Empirical Cumulative Distribution Function (ECDF)

$$F_k = \sum_{j=1}^{k}\pi_j \tag{3}$$

It equals proportion of visits to less than or equal to state k during the time period {1, 2, … M}

Step iv) The frequency of transition from state k to *l* in one step is

$$\pi_{kl} = \sum_{j=1}^{M-1}\delta_{kj}\delta_{l(j+1)} \Big/ \sum_{j=1}^{M}\delta_{kj} \tag{4}$$

It is the ratio of the number of transitions from state k to *l* to the number of visits to state k in the data.

π_{kl} is an estimate of one step transition probability $P\{X_{i+1} \in \text{state } l \mid X_i \in \text{state } k\}$ for the first order chain $\{X\}$. Note that for each state k, $\{\pi_{kl} : l=1,2 \dots N\}$ estimates the transition probability distribution of the next state given that the current state is k. This will be useful in making the prediction of the future state given the present state.

Step v) The frequency of transitions from state h to k and then to *l* in one step is

$$\pi_{hkl} = \sum_{j=1}^{M-2}\delta_{hj}\delta_{k(j+1)}\delta_{l(j+2)} \Big/ \sum_{j=1}^{M-1}\delta_{hj}\delta_{k(j+1)} \tag{5}$$

and it is an estimate of one step transition probability $P\{X_{i+1} \in \text{state } l \mid X_i \in \text{state } k, X_{i-1} \in \text{state } h\}$ for the second order chain $\{Y_i=(X_i,X_{i+1})\}$,

Step vi) If $\pi_{kl} \approx \pi_{hkl}$ for all *h*, then the chain is approximately a first order Markov chain. Otherwise, find the third order transition frequency π_{ghkl}

$$\pi_{ghkl} = \sum_{j=1}^{M-3}\delta_{gj}\delta_{h(j+1)}\delta_{k(j+2)}\delta_{l(j+3)} \Big/ \sum_{j=1}^{M-2}\delta_{gj}\delta_{h(j+1)}\delta_{k(j+2)} \tag{6}$$

and check if $\pi_{ghkl} \approx \pi_{hkl}$ for all *g*. If so, it is approximately a second order Markov chain.

Step vii) The second order chain *Y* is Markov-like but with state space S×S, the Cartesian product of S with itself. This should make the estimate of conditional distribution of the future given the present sharper, thus reduce the level of uncertainty. If necessary one could go to a third or higher order Markov-like chain *Z*, where one records three or more consecutive *X* values instead of two consecutive values.

Step viii) In addition to the above Markov-like modeling, it is also useful to give simple statistical summaries of the data such as the mean, variance and the empirical distribution.

The application of the above procedure to some EAF data is described below.

3. MARKOV-LIKE MODEL APPLICATIONS

Actual EAF data are used to test the models that have just been introduced in section 2. This EAF is a 50 MVA three-phase ac unit which is connected to a 34.5 kV bus behind a specially designed EAF transformer rated at 100 MVA. Twenty seconds of historical arc current and voltage data of phase A is utilized to build the model. A first order Markov-like model is proposed first for description of the concept in section 3.2. Then the second order Markov-like model for accurate modeling and prediction is discussed in section 3.3.

3.1 First Order Markov-like Model

To illustrate, the arc current data are divided into two independent sets. One could compute the estimates of the transition probability matrix from every data set, then use these estimates to simulate and compare the simulation results and the original data with each other to validate the model. The results from the first order Markov-like model are shown in Figs. 2 to 3 and Table 1. The upper part of Fig. 2 is the plots of the Empirical Frequency Function (EFF) π_k for the two actual data sets and their simulation results. Since the data are very close to each other, expanded look within a small interval (states 25 to 35) for these EFFs is shown in the lower part of Fig. 2. Fig. 3 shows the Empirical Cumulative Distribution Function (ECDF) F_k of the corresponding EFFs of Fig. 2. Again, to make the figure clearer, Fig. 3 also gives an expanded look for states 25 to 35 in the lower part. Table 1 indicates the mean and variance indices of the original and simulated current data.

It can be easily seen that the EFFs for the two actual data sets in Fig. 2 are very much alike, and the simulation results are almost the same as the original actual data so that in the figure they are overlapped. In fact, during the simulation, even when the initial condition is changed, the results did not have much variation. It can also be seen that, means are the same for the original and simulation data sets. Their variances are also very close to each other. One may deduce from the simulation results that the transition matrix is a natural parameter of this system. It appears as well that the statistical behaviors of the EAF current are stationary with respect to time from the similarities in characteristics for the two data sets. The statement can also be confirmed by observing the ECDFs of the two data sets along with their simulation results given in Fig. 3.

The same procedure is carried out on the EAF voltage and the corresponding EFFs are shown in Fig. 4. An expanded view is shown for states 25 to 35 in the lower part of this figure, just as in the case of processing EAF current. Since the EFFs are closer to each other than those from the EAF current, the model seems even more accurate for EAF voltage. This is to be expected because the waveform of the EAF voltage is not so irregular as the EAF current. The statistical indices for the two data sets are almost the same, as it is shown in Table 2. Accordingly, specific attention is paid to the EAF current only in the subsequent sections.

3.2 Second Order Markov-like Model

The short-term and long-term predictions of the variable based on the present and past data are of practical interest and value to power engineers. Using the first order Markov-like model, the empirical frequencies of the states help in predicting future averages. Also, the estimated transition probability matrix helps prediction of the immediate future value given the present value. Some parts of the result file consisting of transition matrix are shown in Table 3. Since the matrix is sparse, only nonzero elements are recorded in a format of $\{\pi_{kl}$: its value$\}$. For instance, the first line of data indicates that if the current state is 1, it has a high probability (π_{11}=0.9000) to stay in this state in the next step. This transition matrix is good for a long-term prediction from the earlier analysis, i.e., the simulation results fit with the original data. However, it is found to be not good enough for short-term prediction. For example, in state 41, it has positive probability to enter each of the states 39, 40, 41, 42 and 43. While all of the probabilities are below 0.40, as can be seen from Table 3, one can not predict the next value with accuracy and confidence. The problem of finding a better model for an accurate prediction is now addressed.

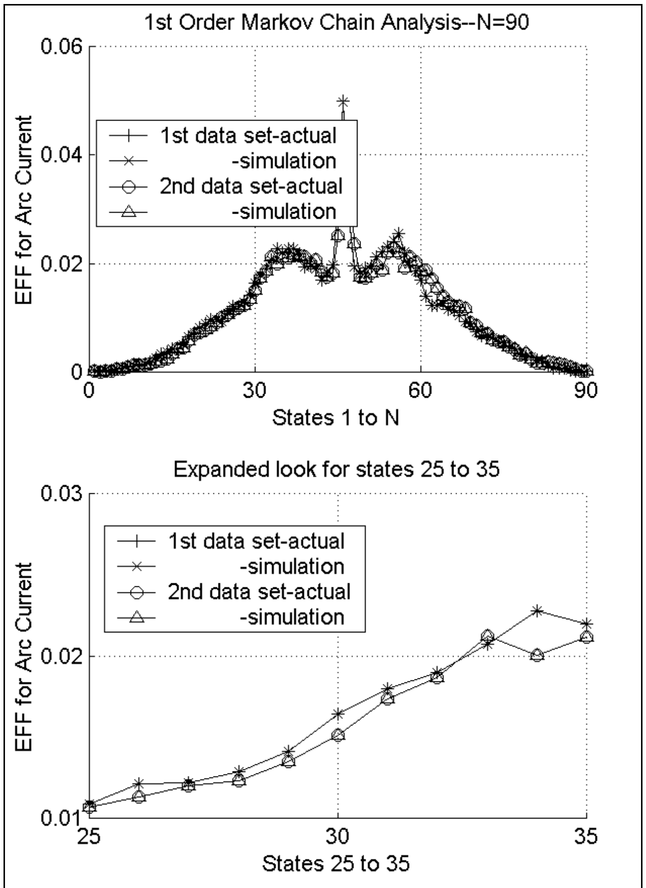

Fig. 2 EFFs for actual arc current from a first order Markov-like model N=90

3.2.1 Concept

For short-term prediction, the transition probability estimates are not sharp (i.e., not very close to 1 or 0) in the first order Markov-like model. This leads one to consider a second order Markov-like model, where a vector $Y_i=(X_{i-1},X_i)$ is recorded for each time point. The first order Markov-like model, as it is shown in the upper part of Fig. 5, does not distinguish between increasing and decreasing trend. On the other hand, a second order Markov-like model does distinguish between increasing and decreasing trend, as shown in the lower part of Fig. 5. Also, most of the transition probabilities are close to 1 or 0, as shown in Table 4. This property makes it effective in short-term prediction, i.e., given current state value Y_i in Y chain, the value of Y_{i+1} can be estimated with higher accuracy.

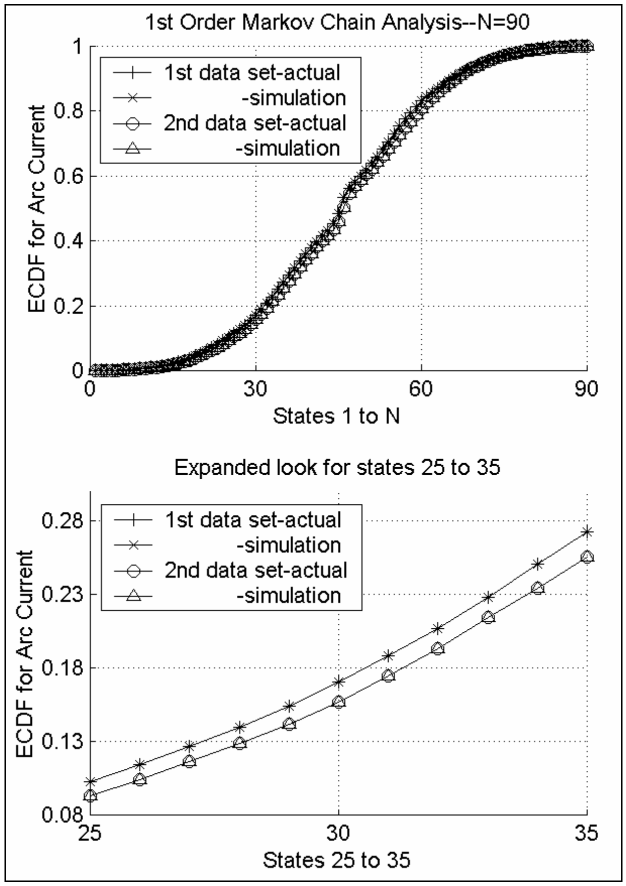

Fig. 3 ECDFs for actual arc current from a first order Markov-like model N=90

Table 1 Statistical indices of the states for original current data and simulation results

Data sets		Mean	Variance
Original data	First set	0.011111	0.000087
	Second set	0.011111	0.000084
Simulation result	First set	0.011111	0.000086
	Second set	0.011111	0.000084

The transition matrix for the second chain $\{Y_i\}$ is similar to that of first order Markov-like model, the dimension is double powered. That is to say, if the state number is set to be 50, for a first order Markov-like model the transition matrix is of the dimension (50×50), while for a corresponding second order Markov-like model, it is of the dimension (50×50)×(50×50)=(2,500×2,500). In Table 4, part of the matrix is also listed in the format of $\{\pi_{hkl}$: its value$\}$, while the chain is defined as $Y_i=(X_{i-1},X_i)$. The results are very satisfactory. Most of transition probabilities are larger than 0.9 or less than 0.1. In many states the probabilities are 1 or 0. Therefore one can predict the value of the arc current at the next step with a high level of confidence. For instance, when the present state is (1,2), one can estimate with reasonable certainty that it will enter state (2,2) according to the transition matrix (vide Table 4). As mentioned earlier, the fact that the estimates of the second order one-step transition probabilities are sharper than those of the first order one-step transition probabilities suggests that the underlying time series is not a first order Markov chain. Nevertheless, this methodology provides accurate prediction of a seemingly chaotic time series.

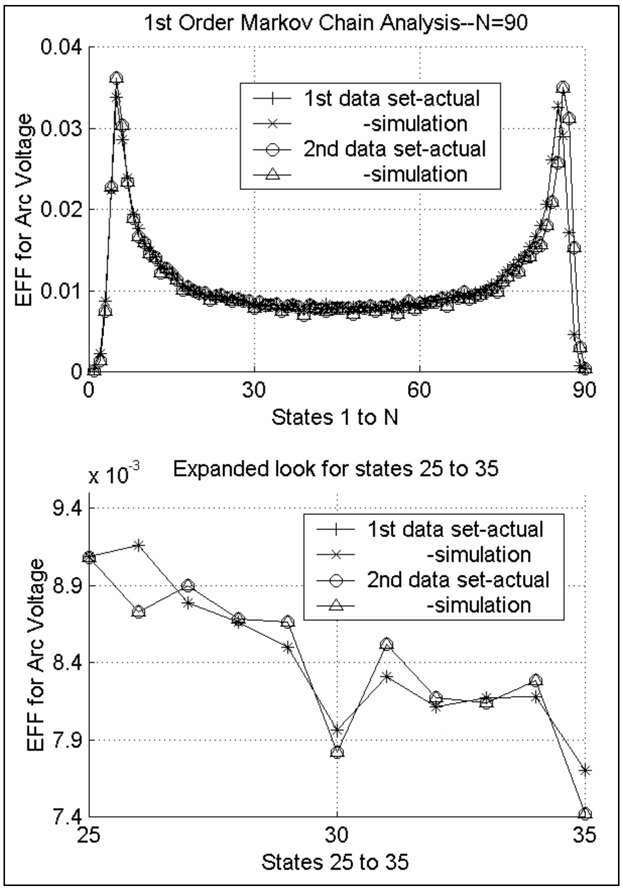

Fig. 4 EFFs for actual arc voltage from a first order Markov-like model N=90

Table 2 Statistical indices of the states for original voltage data and simulation results

Data sets		Mean	Variance
Original data	First set	0.011111	0.000037
	Second set	0.011111	0.000038
Simulation result	First set	0.011111	0.000037
	Second set	0.011111	0.000038

3.2.2 EFF Figures from Data and Simulation

Only the EAF current is processed using a second Markov-like model since it changes more abruptly. The results are shown in Fig. 6. It should be stressed that while the number of the state N is 50 in this case, in a second order Markov-like model, the states have been arranged as the sequence of (1,1), (1,2), … (1,50), (2,1), (2,2),… (50,50). But for convenience, the states in the x-axis are lined up into one dimension from 1 to (50*50)=2500. The data are still divided into two parts in this approach. In Fig. 6, EFFs are plotted for the two actual data sets together with their simulation results derived from a second order Markov-like model processing using transition matrix. In addition, an expanded look for states 500 to 600 is provided in the lower part of Fig. 6.

Table 3 Some elements of transition matrix for a first order Markov-like model

π_{11} : 0.9000 π_{12} : 0.1000
π_{21} : 0.1875 π_{22} : 0.6250 π_{23} : 0.1875
……
$\pi_{41,39}$: 0.0095 $\pi_{41,40}$: 0.3091 $\pi_{41,41}$: 0.3944
$\pi_{41,42}$: 0.2417 $\pi_{41,43}$: 0.0453
……
$\pi_{90,89}$: 0.1052 $\pi_{90,90}$: 0.8948

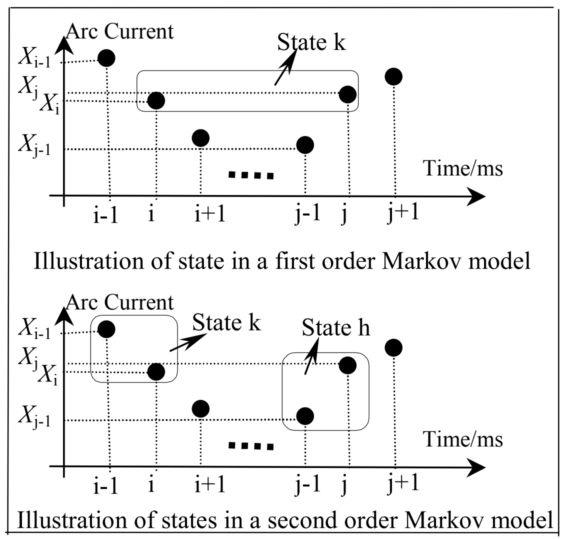

Fig. 5 Comparison of states in first and second order Markov-like model

Table 4 Some elements of transition matrix for a second order Markov-like model

π_{111} : 0.921053 π_{112} : 0.078947
π_{122} : 1.000000
π_{211} : 1.000000
……
$\pi_{27,28,28}$: 0.897181 $\pi_{27,28,29}$: 0.102819
$\pi_{28,26,26}$: 1.000000
……
$\pi_{50,49,49}$: 1.000000
$\pi_{50,50,49}$: 0.076923 $\pi_{50,50,50}$: 0.923077

It may be observed that the EFFs from the two actual data sets and simulation results are really similar. This suggests that EFFs and ECDFs from the actual data show the statistical characteristics when the sample data are large enough. It may not be the case if a small sample of data is selected. Statistical indices such as mean and variance are also included in Table

5. Their similarities in values also show that this model is effective.

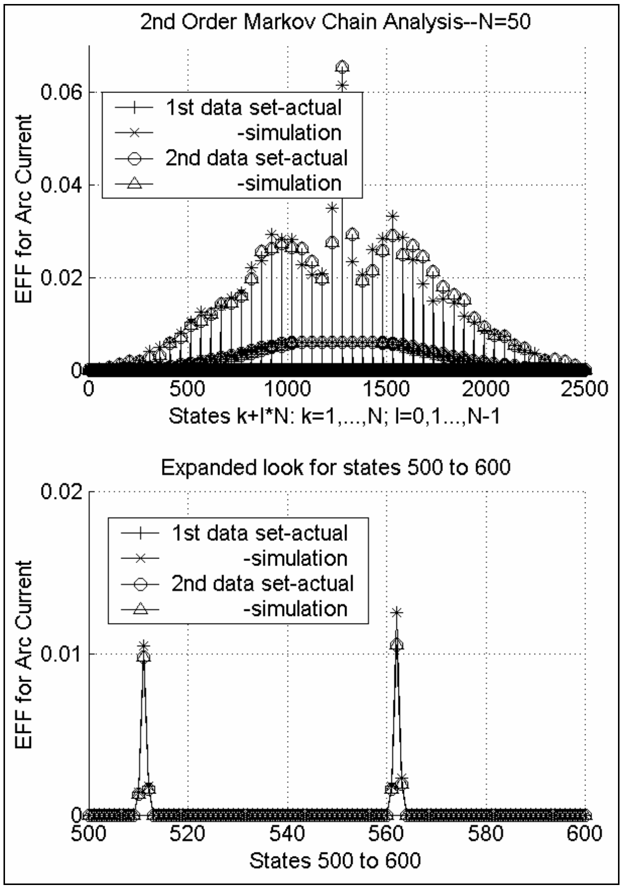

Fig. 6 EFFs for actual arc current from a second order Markov-like model N=50

Table 5 Statistical indices of the states for original current data and simulation results

Data sets		Mean	Variance
Original data	First set	0.0004	0.000007
	Second set	0.0004	0.000008
Simulation result	First set	0.0004	0.000007
	Second set	0.0004	0.000008

Based on these results, it is suggested that the transition matrix is a key parameter of the model for the following reasons:

1) The transition matrix reflects the dynamic characteristics of the system, and it is flexible for prediction.

2) The precision of prediction is high in this model, as it shall be demonstrated in the next section.

4. ONE-STEP-AHEAD PREDICTION

Ten seconds of additional actual data are selected for comparison with the results from one-step-ahead prediction by the second order Markov-like model. When predicting, given the state of $Y_i=(X_{i-1},X_i)$, one can search from the transition matrix and identify the state with maximum conditional probability in the next step. Then the time domain value corresponding to that state is recorded as the estimate $\hat{X}_{i+1}$. The results can be achieved by following the same procedures for ten seconds. Fig. 7 gives 200 points of predicted data along with the actual testing data. It shows that the difference between the actual and predicted data at every time step is very small. A second order Markov-like model is applied in the same way to the predicted data as to the actual data. The EFFs from the result are shown in Fig. 8. These are close to those in Fig. 6. It is also the case in Table 6, where for predicted data the statistical indices of mean and variance are compared with those derived from the actual data. This suggests that the model is very effective for short and long term predictions.

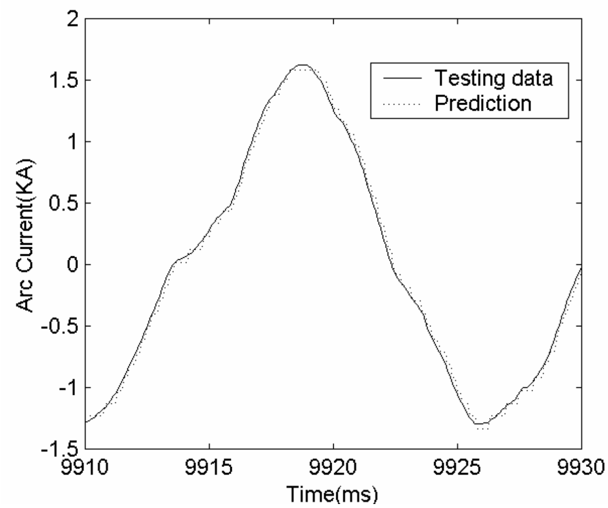

Fig. 7 Waveforms for a part of the predicted and related actual arc current

5. CONCLUSIONS

Based on the Markov-like Modeling of the EAF current/voltage data as detailed in sections 3 and 4, one is able to draw the following conclusions.

i) From a deterministic point of view, the EAF time series $\{X_i$: i=1, 2, … M$\}$ look quite chaotic and nonlinear. However, from the Markov-like modeling point of view, there is a remarkable regularity.

ii) The Empirical Frequency Function, Empirical Transition Frequencies and other time averages exhibit consistency over time. Thus observing the first part of the data set it is possible to make prediction about various time averages for the following data sets. For a long-term time average prediction of this kind, the first order Markov-like model for $\{X\}$ is quite adequate.

iii) In short-term prediction, the second order Markov-like chain Y is better. It is found that the calculated probabilities based on Y are sharp, i.e., very close to 1 or 0. Hence it is better for accurate prediction than the first order ones. This suggests that the first order time series $\{X\}$ is not Markovian and yet the methodology provides a relatively accurate prediction scheme. Thus the procedure is statistically robust. In some cases, it maybe necessary to use a third or higher order Markov-like model, but at the expense of increased computation.

iv) In all, it appears that the Markov-like modeling is a very effective alternative to analyze the dynamics of an EAF system and other kinds of discrete-valued time series with similar behaviors.

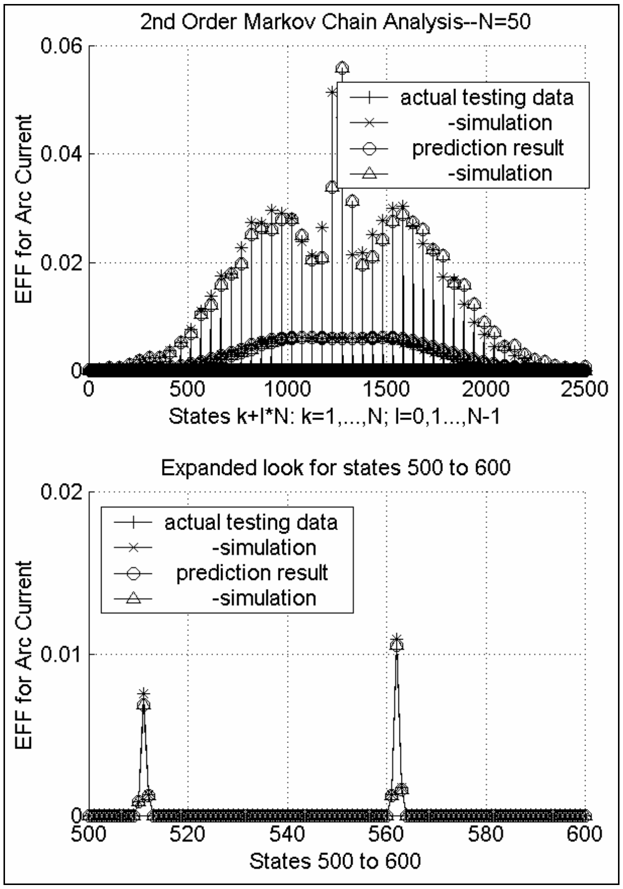

Fig. 8 EFFs for the predicted current from a second order Markov-like model N=50

Table 6 Statistical indices of the states for actual and predicted data

Data sets	Mean	Variance
Predicted data	0.0004	0.000008
Actual data	0.0004	0.000008

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BIOGRAPHIES

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