Line Voltage Drop Calculation in Unbalanced and Distorted Distribution Systems

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Abstract: **The classical problem of three-phase voltage drop calculation, well established in the case of sinusoidal positive sequence system, is extended in the paper with regard to unbalanced and distorted power systems. The adoption of the Park theory permits the introduction of the imaginary power in the voltage drop expression. The imaginary power generalizes the role of the reactive power in the classical treatment and permits to quantifies in one term only the effects on the voltage drop due simultaneously to the load instantaneous stored energy and to the presence of harmonic, interharmonics and sequence components.**

Examples are reported in order to testify, in the case of unbalances and distortions, the effectiveness of the proposed formulation.

Index Terms **- voltage drop, Park transformation, imaginary power, distorted and unbalanced systems, harmonics and interharmonics**

I. INTRODUCTION

The voltage drop is one of the most important quantities in the characterization of transmission and distribution electric power systems. In fact it represents in a certain way the indicator of the effectiveness of the connection between the loads and the generation centers. The voltage drop control is also an essential task both for the stability and the economy of the power system and its calculation, even with the introduction of simplified procedures and approximations, is fundamental for the power system analysis [1].

The voltage drop calculation, in case of ac sinusoidal systems, had rise with the phasor's algebra, then, it was extended from the single-phase case to the three-phase balanced systems. Afterwards, the formulation of the symmetrical component's theory, starting by Stokvis [2], posed the problem of an extension to unbalanced sinusoidal case. Nowadays, the presence of harmonics and interharmonics in power systems point out the need of a new formulation of voltage drop expression [3], that generalizes the former phasorial one but that, in same time, takes into the account the contributions (instantaneous and average) of harmonics, interharmonics and unbalances.

In fact, in the presence of unbalances or/and distortions the accuracy of the voltage drop calculation becomes more weighty because the introduction of some simplifying hypotheses can bring to neglect some disturbance's contributions and then to wrong results.

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The effort of the present paper is to analyze the concept of the voltage drop in three-phase systems under distorted and unbalanced conditions. In order to write the analysis results with the same formalism as the one used for single-phase systems and take into account the simultaneous contributions of the harmonic, interharmonic and sequence components to the voltage drop, the Park transformation is considered. This leads to reconsider the imaginary Park power as an extension of the reactive power concept to the distorted, unbalanced systems also from the point of view of the voltage drop, and not only from that of the line losses. In this respect, the use of the Park quantities is almost mandatory requirement coming from the application of the imaginary power [4,5,6].

II. THEORETICAL BACKGROUND: THE SINGLE-PHASE SINUSOIDAL CASE

Let consider the following complex equation:

$$\overline{V}_m = \overline{V}_v + \left(R + j\omega L\right)\overline{I} = \overline{V}_v + \left(R + jX\right)\overline{I} = \overline{V}_v + \overline{Z}\cdot\overline{I} \qquad (1)$$

which represents a short single-phase line under sinusoidal conditions (fig.1a). The corresponding phasorial diagram is shown in fig.1b. Taking into account the approximate formulation, by Taylor serier expansion of Pitagora's theorem, the phasorial diagram leads to the following relationship for the relative voltage drop:

$$\Delta\widetilde{v} = \frac{V_m - V_v}{V_v} = \frac{\Delta V}{V_v} \qquad (2)$$

In the phasorial diagram of fig.1 ΔV is represented by the magnitude of vector $\overline{OD} - \overline{OA}$ and V_v by the magnitude of $\overline{OA}$. Therefore it can be written as:

$$\Delta\widetilde{v} \overset{\text{Taylor Series}}{\cong} \frac{RP + XQ}{V_v^2} + \frac{1}{2}\frac{I^2\left(X\cos\varphi - R\sin\varphi\right)^2}{V_v^2 + RP + XQ} = f\left(I,\varphi\right) \qquad (3)$$

P and Q being the active and reactive powers respectively, flowing across the right side line terminals. This result can be extended to three-phase circuits in the symmetrical and balanced case: in these conditions, in fact, a single-phase

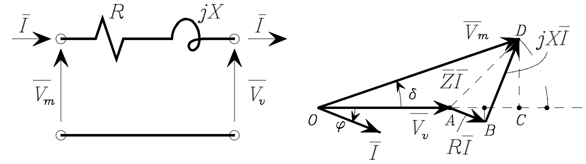

Fig.1. Short single-phase line. (a) Schematic. (b) Phasorial diagram

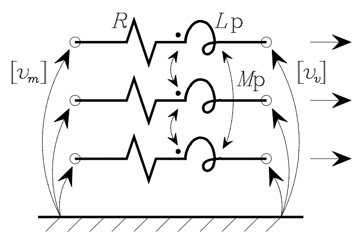

Fig.2. Three-phase line with physical symmetry.

equivalent network can be employed to represent them. Usually, the function (3), in the case of inductive loads and small phase angle δ (fig.1b)[2], gives near the following value:

$$\Delta v \cong \frac{RP + XQ}{V_v^2} \tag{4}$$

Therefore the voltage drop results, in the sinusoidal case, a linear function of the active and reactive load powers.
The meaning of (4) can be fully perceived by assigning a geometrical interpretation to the absolute voltage drop Δv. In fact, this scalar quantity can be considered as the projection of the phasor quantity $\overline{Z} \cdot \overline{I}$ in the direction oriented by the voltage phasor $\overline{V}_V$. It is indeed:

$$\Delta v = \underline{Z} \cdot \underline{I} \bullet \frac{\overline{V}_v}{V_v} = \Re e\left\{\frac{\overline{Z} \cdot \overline{I} \cdot \overline{V}_v^*}{V_v}\right\} = \frac{RP + XQ}{V_v} \tag{5}$$

In this way the usually employed approximated formulation for the voltage drop is obtained.
The employed approach can be immediately extended to three-phase circuits under non-sinusoidal conditions by applying the Fourier series decomposition and Park transformations.

III. The Proposed Method: Park Approach

In the most general case, the time-domain equations for a three-phase distribution line (see Fig.2) are differential equations. For the purposes of this work, they can be more clearly written in term of Heaviside operator [7,8] $\mathrm{p} = d/dt$, so the line impedance vector *z* must be also written in terms of this operator as *z*(p). Therefore, the time-domain equations for the line in Fig.1 can be written as:

$$[v_m(t)] = [v_v(t)] + [z(\mathrm{p})] \cdot [i(t)] \tag{6}$$

By applying the Park transformation[3], the following relationships can be obtained:

$$\begin{cases} \overline{v}_m(t) = \overline{v}_v(t) + \overline{z}\,\overline{i}(t) + \ell \mathrm{p}\,\overline{i}(t) \\ \overline{z} = R + j\dot{\vartheta}\ell = R + jX \\ L - M \overset{\Delta}{=} \ell \end{cases} \tag{7}$$

[2] See appendix A.
[3] See appendix B.

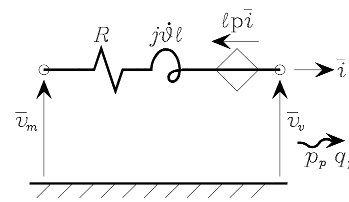

Fig.3. Park representation of a two-port network representing a three-phase short line.

These equations are the Park equations describing the two-port network shown in Fig.3. They differ from those describing a single-phase line under sinusoidal conditions because of the presence of the dynamic term $\ell \mathrm{p}\bar{i}(t)$.
It is known [5] that the application of the Park transformation allows for extending the phasor formalism adopted for the sinusoidal case to the non-sinusoidal one by substituting the instantaneous Park vectors to the phasor and the instantaneous rms value of the Park vector (Appendix A) to the sinusoidal quantities. Applying the rms instantaneous Park voltages, the relative voltage drop can be written as:

$$\Delta v(t) = \frac{\left|\overline{v}_m(t)\right| - \left|\overline{v}_v(t)\right|}{\left|\overline{v}_v(t)\right|} = \frac{v_m(t) - v_v(t)}{v_v(t)} \tag{8}$$

The formal extension of the geometrical approach typical of the sinusoidal case in three-phase systems leads - for small values of the phase angle δ (Fig.4) – to the following instantaneous rms voltage drop:

$$\begin{aligned} \Delta v(t) &= \frac{(\overline{z} + \ell \mathrm{p})\overline{i}(t)}{v_v(t)} \bullet \frac{\overline{v}_v(t)}{v_v(t)} = \Re e\left\{\overline{z} \cdot \overline{i}(t)\frac{\overline{v}_v^*(t)}{v_v^2(t)} + \left(\ell \mathrm{p}\overline{i}(t)\right)\frac{\overline{v}_v^*(t)}{v_v^2(t)}\right\} = \\ &= \frac{Rp_P(t) + Xq_P(t)}{v_v^2(t)} + \frac{\ell \mathrm{p} p_P(t)}{v_v^2(t)} - \frac{\ell\, \overline{i}(t) \bullet \mathrm{p}\overline{v}_v(t)}{v_v^2(t)} \end{aligned} \tag{9}$$

where $p_p(t) = \Re e\{\overline{v}_v(t) \cdot \overline{i}^*(t)\}$ is the instantaneous Park real power, and $q_p(t) = \Im m\{\overline{v}_v(t) \cdot \overline{i}^*(t)\}$ is the instantaneous Park imaginary power [4,5,6].
The following considerations apply to the expression (9).

a) The first term represents the instantaneous voltage drop evaluated by means of an equation that is formally the same as for the single-phase case. Despite the similarity is once again formal, due to the fact that the instantaneous power $p_p(t)$ is considered instead of the active power *P*, and the instantaneous imaginary power $q_p(t)$ is considered instead of the reactive power *Q*, this term keeps its meaning extending it to the non-sinusoidal unbalanced condition. This extension is due to the presence of the

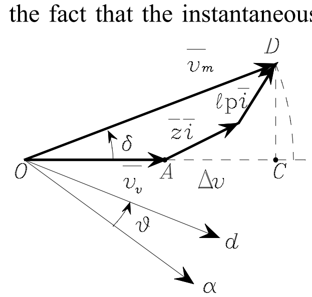

Fig.4. Graphic representation of the drop voltage in the stationary αβ plane and in the rotating dq one.

imaginary power, that is a dynamic term and is related not only to dielectric and magnetic phenomena but also to harmonic and sequence components [4,5,6,10].

b) The second term is connected to the rate of variation of the load instantaneous power. This term gives further evidence that the optimal transmission is characterized by $p_P(t)=P=$constant which is a specific condition of the balanced three-phase systems.

c) The third term is related to the rate of variation of the voltage projected on the direction identified by the current Park vector. This term is nil when the stationary axes voltage Park vector describes a circumference with constant speed. This condition occurs when the supply voltages are sinusoidal and belong to the positive sequence only.

The approximation introduced by using (9) deduced by graphic approximations instead of (8) can be estimated and depends on the ratio between the line and load sequence inductance value, in a similar way as that indicated for the sinusoidal single-phase case, as shown in [9].

Equation (9) is valid under any condition. This gives to equation (9) the possibility to be applied to many systems condition, as for example the presence of static converters and non-linear loads.

IV THE EFFECT OF HARMONIC AND SEQUENCE COMPONENTS ON THE VOLTAGE DROP UNDER NON-SINUSOIDAL CONDITIONS

On this line, the definition of the role performed by harmonics and sequence components present in the network becomes very important on the application point of view. Equation (9) can be reconsidered in order to clarify the dependence of $p_p(t)$, $q_p(t)$, $\bar{i}(t)$, $\bar{v}_v(t)$ on the harmonic and sequence components. Observing that [4,5]:

$$\begin{cases} \overline{w}(t)=\sum_{h=-\infty}^{+\infty}\overline{w}_h(t)=\sum_{h=-\infty}^{+\infty}\overline{W}_h e^{j(h\omega t-\vartheta(t))} \\ \overline{a}_P(t)=\overline{v}(t)\cdot\overline{i}^*(t)=p_P(t)+jq_P(t)=\sum_{h=-\infty}^{+\infty}\overline{V}_h\overline{I}_h^*+\sum_{\substack{-\infty\\h\neq k}}^{+\infty}\overline{V}_h\overline{I}_k^* e^{j(h-k)\omega t} \end{cases} \tag{10}$$

the different terms in (4) can be rewritten as:

$$\begin{cases} \dfrac{Rp_P(t)+Xq_P(t)}{v_v^2(t)}=\left\{R\cdot\Re e\left[\sum_{h=-\infty}^{+\infty}\overline{V}_h\overline{I}_h^*+\sum_{\substack{-\infty\\h\neq k}}^{+\infty}\overline{V}_h\overline{I}_k^* e^{j(h-k)\omega t}\right]+\right. \\ \qquad\left.+X\cdot\Im m\left[\sum_{h=-\infty}^{+\infty}\overline{V}_h\overline{I}_h^*+\sum_{\substack{-\infty\\h\neq k}}^{+\infty}\overline{V}_h\overline{I}_k^* e^{j(h-k)\omega t}\right]\right\}\Big/\sum_{h=-\infty}^{+\infty}V_h^2 \\ \dfrac{\ell p p_P(t)}{v_v^2(t)}=\ell\cdot\Re e\left\{\sum_{\substack{-\infty\\h\neq k}}^{+\infty}\left(j(h-k)\omega\right)\overline{V}_h\overline{I}_k^* e^{j(h-k)\omega t}\right\}\Big/\sum_{h=-\infty}^{+\infty}V_h^2 \\ \dfrac{\ell\,\overline{i}(t)\bullet p\overline{v}_v(t)}{v_v^2(t)}=\left\{\ell\cdot\sum_{h=-\infty}^{+\infty}-\left(h\omega-\dot{\vartheta}\right)Q_h+\right. \\ \qquad\left.+\Re e\left\{\sum_{\substack{-\infty\\h\neq k}}^{+\infty}\left(-j\,\ell\left(k\omega-\dot{\vartheta}\right)\right)\overline{I}_h\overline{V}_h^* e^{j(h-k)\omega t}\right\}\right\}\Big/\sum_{k=-\infty}^{+\infty}V_h^2 \end{cases} \tag{11}$$

This result formally unifies the harmonic and sequence effects in three-phase circuits since it takes into account both the harmonic and sequence components as the components of the generalized Fourier series decomposition of the Park vector. In fact, each harmonic component k belongs to the positive sequence if $k>0$ or to the negative sequence if $k<0$ [5]. It presents the most general formulation of the voltage drop in physically symmetrical three-phase networks under non-sinusoidal condition. Moreover it confirms the presence of pulsating component on the voltage drop, giving evidence of the harmonic and sequence additional contribution with respect to the sinusoidal, positive sequence, balanced three-phase case.

A. Voltage drop average value and its connection with the classical expression

Starting from the above general expression of the voltage drop in time domain, it is possible to evaluate the following average value with reference to the interval [t, $t+T$]:

$$\langle\Delta v\rangle=\frac{1}{T}\int_t^{t+T}\Delta v(t)dt \tag{12}$$

This calculation is justified by the importance of the voltage rms value on the loads and distribution system operation.
Applying (7) to the formula (6), it is possible to obtain:

$$\langle\Delta v\rangle=\frac{R\cdot\sum_{h=-\infty}^{+\infty}P_h+X\cdot\sum_{h=-\infty}^{+\infty}Q_h}{\sum_{h=-\infty}^{+\infty}V_h^2}-\frac{\ell\cdot\sum_{h=-\infty}^{+\infty}-\left(h\omega-\dot{\vartheta}\right)Q_h}{\sum_{k=-\infty}^{+\infty}V_h^2} \tag{13}$$

thus:

$$\langle \Delta v \rangle = \frac{R \cdot \sum_{h=-\infty}^{+\infty} P_h + \omega\ell \cdot \sum_{h=-\infty}^{+\infty} hQ_h}{\sum_{h=-\infty}^{+\infty} V_h^2} \tag{14}$$

The unbalanced sinusoidal case is of particular interest. In these conditions the presence of contributions associated to positive and negative sequences only brings to the following:

$$\langle \Delta v \rangle = \frac{R \cdot (P_1 + P_2) + \omega\ell \cdot (Q_1 - Q_2)}{V_1^2 + V_2^2} \tag{15}$$

In (10) it is possible to recognize a generalization of the classical expression typical of the presence of the positive sequence only.
In this case the reactive power – represented as maximum value in classical theory – appears as the average value of the imaginary power [4,5].

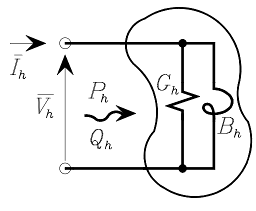

Fig.5. Load representation.

Referring P_h, Q_h and V_h present in (15) to the load representation shown in Fig.5, we obtain:

$$\begin{cases} V_h^2 = k_h V_1^2 \\ P_h = V_h^2 G_C \\ Q_h = V_h^2 B_C / h \end{cases} \tag{16}$$

And (9) becomes:

$$\langle \Delta v \rangle = \frac{RP_1 \cdot \sum_{h=-\infty}^{+\infty} k_h + \omega\ell Q_1 \cdot \sum_{h=-\infty}^{+\infty} k_h}{V_1^2 \cdot \sum_{h=-\infty}^{+\infty} k_h} = \frac{RP_1 + \omega\ell Q_1}{V_1^2} \tag{17}$$

This result shows that the voltage drop calculated for the fundamental harmonic component only represents the generalized average value of the voltage drop expressed in (10).
The obtained results suggest in addition the following comments. Concerning the size of system components, the use of the classical expression:

$$\Delta V = \frac{RP + \omega\ell Q}{V_v} \tag{18}$$

is confirmed. The important role is, in this case, assumed by the reactive power associated to the fundamental harmonic positive sequence. As concerns the compensation and stability [12] problems – that rely to dynamic topics – the use of the formula (10) derived from the Park approach is more appropriated giving results on time domain. In this case the imaginary power $q_p(t)$ takes the role of the reactive power Q.

V. The effect of interharmonic components on the voltage drop

It is particularly interesting the case in which interharmonics are present in the power system In this case the general formulations (11,13,14) are still valid. Also the equations developed concerning the periodic conditions are valid when frequency, harmonic order and time period are referred to the basic quantities (i.e. the ones associated to the basic frequency of the correspondent waveform) instead of the network frequency (50 or 60 Hz).
In this case the Park quantities become:

$$\begin{cases} \overline{w}(t) = \sum_{h_F=-\infty}^{+\infty} \overline{w}_{h_F}(t) = \sum_{h_F=-\infty}^{+\infty} \overline{W}_{h_F} e^{j(h_F \omega_F t - \vartheta(t))} \\ \omega_F = \frac{2\pi}{T_F} = 2\pi f_F \end{cases} \tag{14}$$

where:

- f_F is the Fourier's basic frequency (it is the greatest common divisor of all the frequencies components in the signal);
- h_F is the harmonic order referred to f_F;
- T_F is the period associated to the basic frequency.

Furthermore the relation between these latter quantities and the ones related to the network frequency:

$$\frac{h_F}{h} = \frac{f_F}{f} = \frac{T}{T_F} \tag{15}$$

VI Some examples

Some typical examples are here considered that refer to a short line supplying a passive load. The different examples differ for the voltage and current waveforms imposed at the output port.
In particular, the following situations are considered.

a) The voltages imposed at the line output port are sinusoidal and belong to the positive sequence; the load draws sinusoidal currents of both positive and negative sequence ($I_2 = 0.25 \cdot I_1$). The voltages at the line-input port are obtained by means of (7). The considered circuit is shown in Fig.6a.
b) The voltages imposed at the line output port are 6-step voltages with basic frequency 50Hz. The current drawn by the passive load is consequently unbalanced and distorted. The voltages at the line-input port are obtained by means of (7). The considered circuit is shown in Fig.7a.
c) The voltages imposed at the line output port are periodical with subharmonic (f_s=20Hz, V_s=0.1V) and interharmonic (f_i=320Hz, V_s=0.2V) components, the basic component have f=50 Hz with magnitude V. The current drawn by the passive load is consequently unbalanced and distorted. The voltages at the line-input port are obtained by means of (7). The considered circuit is shown in Fig.8a.

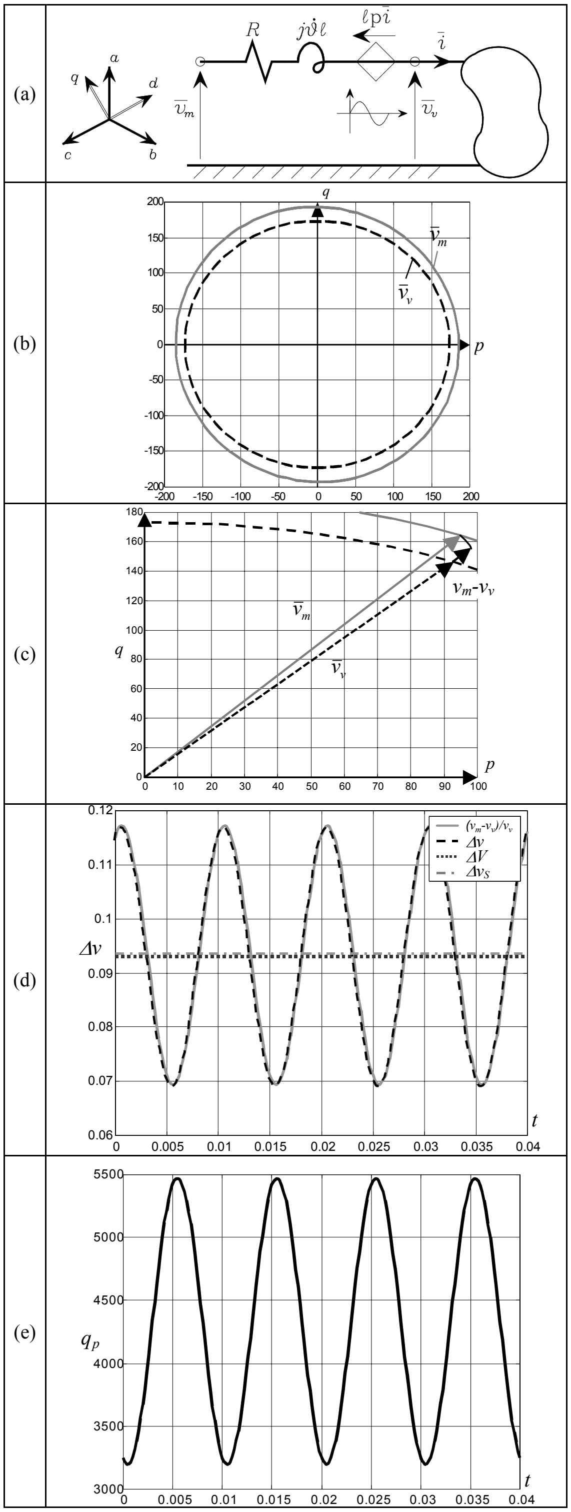

Fig.6. Numerical example where the voltages at the line output port are sinusoidal and belong to the positive sequence. (a) Circuit representation. (b) Polar diagram of Park voltages. $\bar{v}_v$: Dashed line; $\bar{v}_m$: solid line. (c) Enlargement of a significant portion of (b) diagram. (d) Voltage drop waveform given by (8), (9), (18) and by the symmetrical components theory. (e) Park imaginary power waveform

The following diagrams show the results of the above numerical simulation.

- Figures 6b, 7b and 8b show the polar diagrams of the Park vectors of the voltages at the input and output ports of the line. Figures 6c, 7c and 8c show the enlargements of a significant portion of the above diagrams, which stress the instantaneous voltage drop evaluated by (7) specifically expressed in terms of Park variables.
- Figures 6d, 7d and 8d show the voltage drop waveforms evaluated by means of the usual definition (8), the proposed relationship (9) and the classical formulation (18).
- Figures 6e, 7e and 8e show the Park imaginary power waveform.

The above mentioned diagrams show that:

1) The polar diagrams can be seen as an extension, in the {d,q,o} domain, of the phasorial diagrams typical of the sinusoidal single-phase conditions. They give evidence of the voltage drop in three-phase circuits in a much clearer way than the traditional approach, which works only under balanced symmetrical conditions.
2) The comparison of the diagrams in Figures 6d, 7d and 8d proves that the proposed algorithm, based on the Park approach, is correct, since the voltage drop diagram is the same as that computed using (8).
3) The exam of the diagrams in Figures 6d, 7d and 8d confirms the implication typical of classical theory: the classical voltage drop is equal to the mean value of the Park voltage drop.
4) The diagrams of the Park imaginary power show that this quantity has fairly the same waveform as the instantaneous voltage drop, and hence give evidence of the role of this quantity that is completely disregarded by the classical theory.

In the case of examples b) and c), that are studies referred to the harmonic distortion component (see Fig.7) and to the interharmonic ones (see Fig.8), respectively, a comparison is made with the results obtained by means of commercial software on power systems harmonics [13].

The simulation with this software brings to the point-dashed lines Δv_H in Figg.7d and 8d. These results represent the mean value of the voltage drop as seen by Park approach and then the classical voltage drop. This result put in evidence how the Park approach is the most general one. In fact, in addition to the general solution, the Park application can easily bring, through the evaluation of the mean value, to the classical voltage drop evaluation.

VII. CONCLUSIONS

The usual formulation of the voltage drop, generally limited to the sinusoidal balanced case, was extended to the most general non-sinusoidal unbalanced case. This extension gives evidence of the effects of the harmonic and sequence components. In particular, the analysis in terms of the Park vector confirms the single-phase nature of the Park variables and stresses the role of the Park imaginary power. This last quantity appears to be a generalization of the reactive power concept to any operating condition as far as the voltage drop is considered.

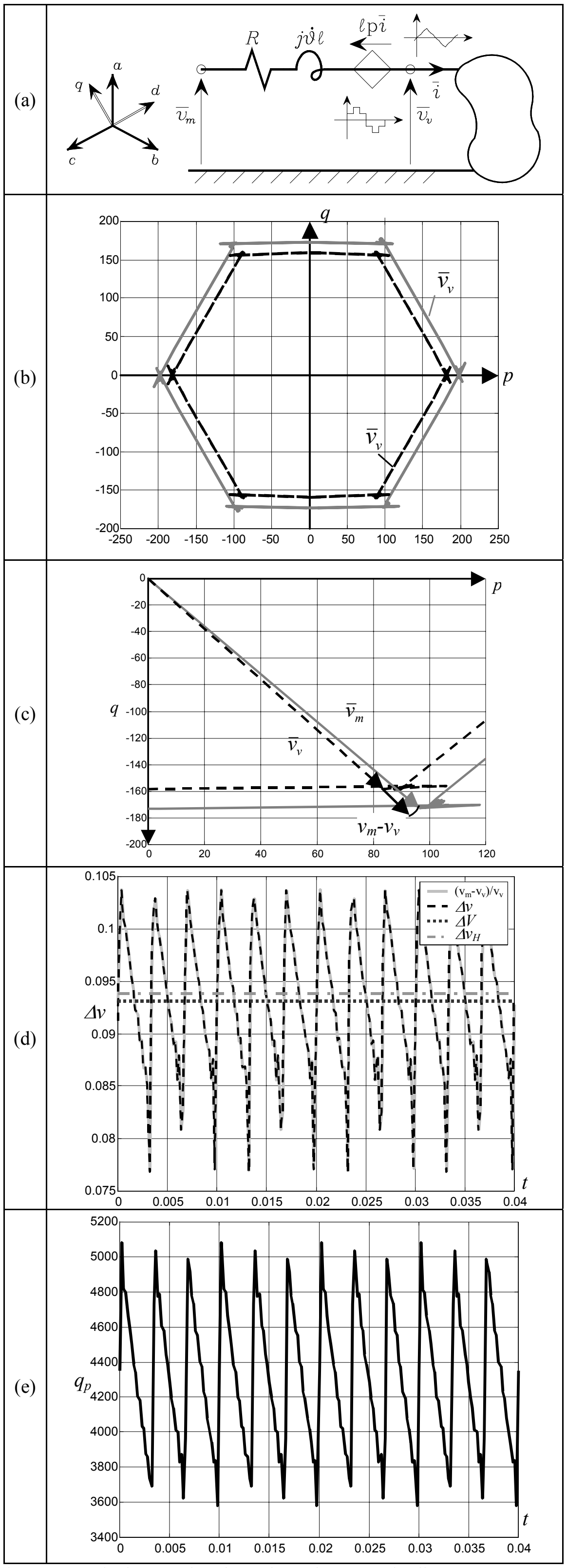

Fig.7. Numerical example where the voltages at the line output port are 6-step waveform. (a) Circuit representation. (b) Polar diagram of Park voltages. $\bar{v}_v$: Dashed line; $\bar{v}_m$: solid line. (c) Enlargement of a significant portion of (b) diagram. (d) Voltage drop waveform given by (8), (9), (18) and by a commercial software. (e) Park imaginary power waveform.

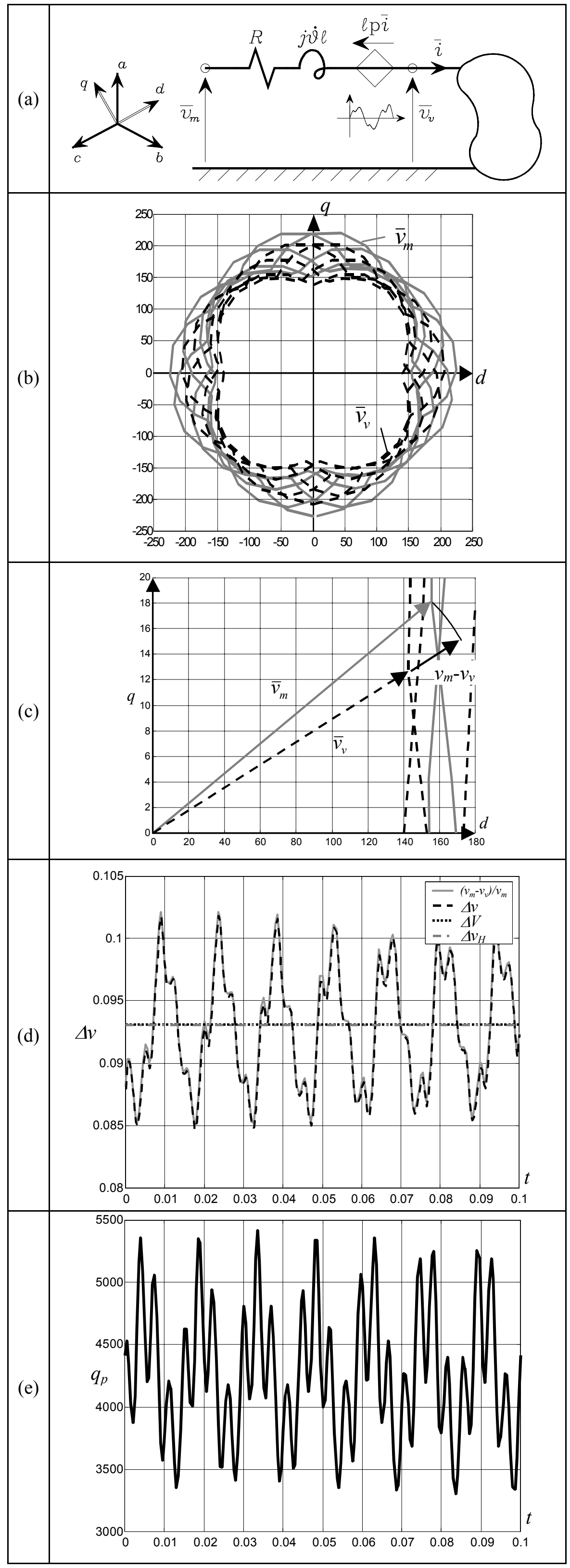

Fig.8. Numerical example where the voltages at the line output port are periodical with subharmonic and interharmonic components. (a) Circuit representation. (b) Polar diagram of Park voltages. $\bar{v}_v$: Dashed line; $\bar{v}_m$: solid line. (c) Enlargement of a significant portion of (b) diagram. (d) Voltage drop waveform given by (8), (9), (18) and by a commercial software. (e) Park imaginary power waveform.

This discloses new opportunities to the study of active compensators that can be usefully employed also to control the voltage drop. At last, it is important to underline the role played by the terms $p p_p(t)$, $p\,\overline{v}_v(t)$: they not only emphasize how synthetic and powerful the Park formalism is, but also confirm the advantage, for energy transmission, to employ a three-phase sinusoidal, symmetric and balanced system.

The obtained results suggest in addition the following comments. Concerning the size of the system components, the use of the classical expression (18) is confirmed. As concern the compensation and stability problems the use of the (9) derived from Park approach is more appropriated giving results on time domain. In this case the imaginary power $q_p(t)$ takes the role of the reactive power Q.

Moreover, since active compensators have been widely studied and are presently available to compensate the imaginary Park power, the proposed analysis also shows a method for practical control of the voltage drop.

APPENDIX A.

The error introduced by using the graphic formulation (5) instead of (2) for evaluating the voltage drop, depends on the ratio between the line and load sequence impedance value as documented in [9].

This error can be evaluated in terms of relative deviation by using the following relationship:

$$\varepsilon\% = \left(\Delta\tilde{v} - \Delta v\right)/\,\Delta\tilde{v}\cdot 100 \qquad \text{(A1)}$$

Fig.9 shows the relative deviation $\varepsilon\%$ as a function of the power factor $\cos\varphi$ and of the ratio between the line sequence impedance Z and load sequence impedance Z_L.

Taking into account that the usual value of Z/Z_L in practical cases is less then 0.05 [14], the relative deviation $\varepsilon\%$ is less than 0.5%, the angle δ is less than 2 degrees and the voltage drop is less than 5%.

A relative deviation $\varepsilon\%$ of 2% implies an inductance ratio of 0.25, then a voltage drop (3) close to 27%, a not realistic value for a correct transmission or distribution line design.

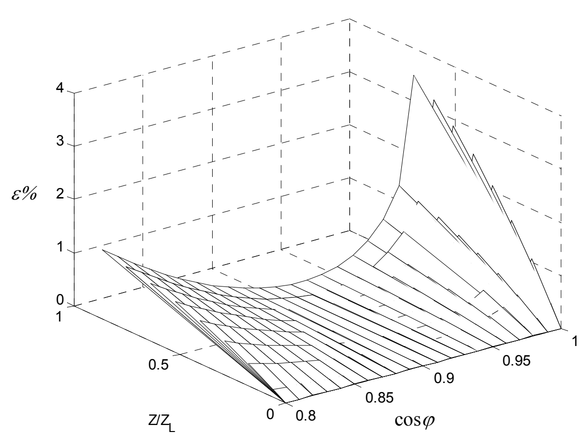

Fig.9. Relative deviation as a function of the ratio Z/Z_L and of the power factor $\cos\varphi$.

APPENDIX B. PARK TRANSFORMATION APPROACH

If the following Park transformation T [2,3] is employed:

$$\begin{cases} \left[T\right] = \sqrt{\dfrac{2}{3}}\cdot\begin{bmatrix} \cos\vartheta_1 & \cos\vartheta_3 & \cos\vartheta_2 \\ -\sin\vartheta_1 & -\sin\vartheta_3 & -\sin\vartheta_2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \\ \vartheta_k = \vartheta + \left(k-1\right)\dfrac{2\pi}{3} \qquad k = 1,2,3 \end{cases} \qquad \text{(B1)}$$

(B1) can be expressed by means of the {d,q,o} variables. Then the Park vectors are defined as (Fig.10):

$$\begin{aligned} \overline{w}(t) &= w_d(t) + j w_q(t) = \\ &= \sqrt{\frac{2}{3}}\left(w_a(t) + \overline{\alpha} w_b(t) + \overline{\alpha}^2 w_c(t)\right)e^{-j\vartheta(t)} = \overline{w}_{\alpha\beta}e^{-j\vartheta(t)} \end{aligned} \qquad \text{(B2)}$$

where $\overline{\alpha} = e^{j2\pi/3}$.

The square of the instantaneous rms value of the Park vector can be written in the following form:

$$\begin{aligned} \overline{w}(t)\overline{w}^*(t) &= w_d^2(t) + w_q^2(t) = \overline{w}_{\alpha\beta}(t)\overline{w}^*_{\alpha\beta}(t) = \\ &= \frac{2}{3}\left[w_a^2(t) + w_b^2(t) + w_c^2(t)\right] \end{aligned} \qquad \text{(B3)}$$

that is invariant with the axis choice. The axis can be fixed, {α,β}, or rotating at speed $\dot{\vartheta}$, {d,q} (Fig.10). Equation (B3) is a formal time-domain generalization of the rms three-phase value under sinusoidal condition.

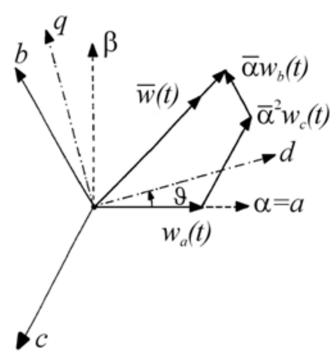

Fig.10. Geometric interpretation of Park transformation.

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